

MATH 1650: SECTIONS 5.2 AND 5.3: FUNCTION ARITHMETIC

DEFINITION: Suppose f and g are functions and x is in both the domain of f and the domain of g .

- The **sum** of f and g , denoted $f + g$, is the function defined by the formula

$$(f + g)(x) = f(x) + g(x)$$

That is, to add functions, we add their outputs.

- The **difference** of f and g , denoted $f - g$, is the function defined by the formula

$$(f - g)(x) = f(x) - g(x)$$

That is, to subtract functions, we subtract their outputs.

- The **product** of f and g , denoted fg , is the function defined by the formula

$$(fg)(x) = f(x)g(x)$$

That is, to multiply functions, we multiply their outputs.

- The **quotient** of f and g , denoted $\frac{f}{g}$, is the function defined by the formula

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad \text{provided } g(x) \neq 0.$$

That is, to divide functions, we divide their outputs.

DEFINITION: If x is in the domain of f and $f(x)$ is in the domain of g :

- the **composite** of g with f , $g \circ f$ is the function defined by the formula

$$(g \circ f)(x) = g(f(x))$$

That is, $g \circ f$ is the function that does f to x then does g to $f(x)$.

EXAMPLE: Let $f = \{(0, 3), (1, 2), (2, -5)\}$ and $g(t) = t - 7$. Find the following:

- $(f + g)(1)$

- $(f - g)(0)$

- $(fg)(0)$

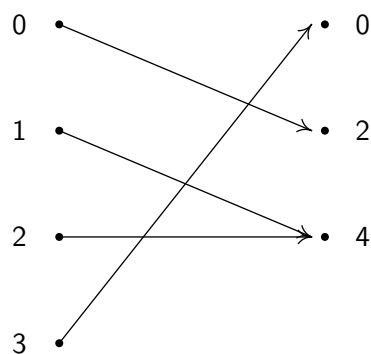
- $\left(\frac{g}{f}\right)(2)$

- $(g \circ f)(0)$

- $(f \circ g)(7)$

EXAMPLE: Suppose f and g are described below, find the following:

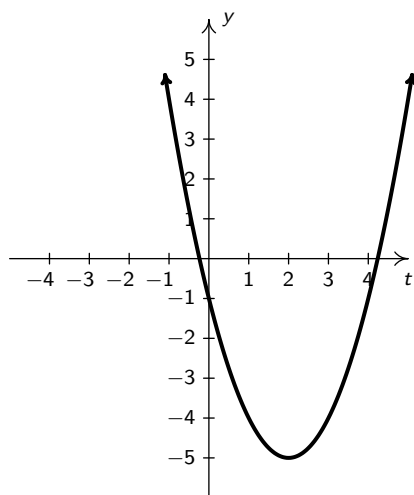
The function f described below:



- $(f + g)(2)$

- $(g \circ f)(0)$

The function g whose graph is below:



- $(fg)(0)$

- $(f \circ f)(3)$

EXAMPLE: Let $f(x) = 3x - x^2$. Find and simplify $\frac{f(x+h) - f(x)}{h}$.

EXAMPLE: Find and simplify expressions for $(g \circ f)(x)$ and $(f \circ g)(x)$. List the domains for $g \circ f$ and $f \circ g$.

REMEMBER: Find the domain **before** you simplify!

- $f(x) = x^2, g(x) = \sqrt{x}$

- $f(x) = \frac{2x}{x-1}$ and $g(x) = \frac{4}{x+1}$

EXAMPLE: Let $h(x) = 4x - x^2$. Find functions f and g so that $h = f - g$.

EXAMPLE: Let $h(z) = \sqrt{4 - z}$. Find functions f and g so that $h = g \circ f$.

EXAMPLE: Let $r(t) = \frac{3-t}{t^2+1}$. Find functions f and g so that $r = \frac{f}{g}$.

EXAMPLE: Let $R(t) = \sqrt{\frac{3-t}{t^2+1}}$. Find functions f , g , and h so that $R = h \circ \left(\frac{f}{g}\right)$.

HOMEWORK: Section 5.2: 1 - 73 every other odd; Section 5.3: 1 - 65 every other odd.